SIDDARTHA INSTITUTE OF SCIENCE AND TECHNOLOGY::PUTTUR

(AUTONOMOUS)

Siddharth Nagar, Narayanavanam Road – 517583

MODEL QUESTION BANK (DESCRIPTIVE)

Subject with Code : DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS **(19HS0831)**

Course & Branch: B.Tech – CE,EEE,ME & ECE.

Year & Sem: I-II **Regulation:** R19

UNIT –I
(First and higher order Ordinary Differential Equations)

1) a) Solve $(2x - y + 1)dx + (2y - x - 1)dy = 0$ [6M]

   b) Solve $(y^2 - 2xy)dx + (2xy - x^2)dy = 0$ [6M]

2) a) Solve $\frac{dy}{dx} + \frac{ycosx+siny+y}{sinx+xcosy+x} = 0$ [6M]

   b) Solve $(x^2 - ay)dx = (ax - y^2)dy$ [6M]

3) a) Solve $x\frac{dy}{dx} + y = logx.$ [6M]

   b) Solve $\frac{dy}{dx} + 2xy = e^{-x^2}$ [6M]

4) a) Solve $(1 + y^2)dx = (tan^{-1}y - x)dy.$ [6M]

   b) Solve $(x + 1)\frac{dy}{dx} - y = e^{3x}(x + 1)^2.$ [6M]

5) a) Solve $x\frac{dy}{dx} + y = x^3y^6.$ [6M]

   b) Solve $\frac{dy}{dx} + y.tanx = y^2\,secx$ [6M]

6) a) Solve $(D^2 + 5D + 6)y = e^x$ [6M]

   b) Solve $(D^2 - 4D + 3)y = 4e^{3x}$ given ; $y(0) = -1, y^1(0) = 3.$ [6M]

7) a) Solve $(D^2 - 3D + 2)y = cos3x$ [6M]

   b) Solve $(D^2 - 4D)y = e^x + sin3x.cos2x$ [6M]

8) a) Solve $(D^2 + 4D + 4)y = 4cosx + 3sinx$ [6M]

   b) Solve $(D^2 + 1)y = sinx.sin2x$ [6M]

9) a) Solve $(D^2 + D + 1)y = x^3$ [6M]

   b) Solve $(D^2 - 3D + 2)y = xe^{3x} + sin2x$ [6M]

10) a) Solve $(D^2 + 4)y = e^x + sin2x + cos2x.$ [6M]

    b) Solve $(D^2 + 4D + 3)y = e^{-x}sinx + x.$ [6M]

UNIT –II

(Equations reducible to Linear Differential Equations)

1) a) Solve $(D^2+a^2)y=\tan ax$ by method of variation of parameters. [6M]

b) Solve $(D^2-2D)y=e^x\sin x$ by method of variation of parameters. [6M]

2) a) Solve $(D^2+4)y=Sec2x$ by method of variation of parameters. [6M]

b) Solve $(D^2+1)y=Co\sec x$ by method of variation of parameters. [6M]

3) a) Solve $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\log x$ [6M]

b) Solve $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^4$. [6M]

4) a) Solve $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}=\frac{12\,logx}{x^2}$ [6M]

b) Solve $(x^2D^2-4xD+6)y=x^2$ [6M]

5) Solve $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=sin2[\log(1+x)]$ [12M]

6) Solve $(1+x)^2\frac{d^2y}{dx^2}-3(1+x)\frac{dy}{dx}+4y=x^2+x+1$ [12M]

7) a) Solve $\frac{dx}{dt}=3x+2y$; $\frac{dy}{dt}+5x+3y=0.$ [6M]

b) Solve $\frac{dy}{dx}+y=z+e^x$; $\frac{dz}{dx}+z=y+e^x.$ [6M]

8) Solve $\frac{dx}{dt}+2x+y=0$; $\frac{dy}{dt}+x+2y=0; gien\ x=1\ and\ y=0\ when\ t=0$ [12M]

9) An uncharged condenser of capacity is charged applying an e.m.f $E\ sin\frac{t}{\sqrt{LC}}$ through leads of self-inductance L and negligible resistance. Prove that at time ‘t’, the charge on one of the plates is $\frac{EC}{2}\left[sin\frac{t}{\sqrt{LC}}-\frac{t}{\sqrt{LC}}cos\frac{t}{\sqrt{LC}}\right]$. [12M]

10) Find the current ‘i’ in the LCR circuit assuming zero initial current and charge i.

If R=80 ohms, L=20 henrys, C=0.01 farads and E=100 V. [12M]

UNIT –III
(Partial Differential Equations)

1) a) Form the Partial Differential Equation by eliminating the constants from
$2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$. [6M]

b) Form the Partial Differential Equation by eliminating the constants from
$(x-a)^2 + (y-b)^2 = z^2 cot^2\alpha$. where '$\alpha$' is a parameter. [6M]

3) a) Form the Partial Differential Equation by eliminating the constants from
$z = a.\,log\left[\frac{b(y-1)}{(1-x)}\right]$. [6M]

b) Form the Partial Differential Equation by eliminating the constants from
$\log(az-1) = x + ay + b$. [6M]

4) a) Form the Partial Differential Equation by eliminating the arbitrary functions from
$z = f(x^2 - y^2)$. [6M]

b) Form the Partial Differential Equation by eliminating the arbitrary functions from
$z = f(x) + e^y.\,g(x)$ [6M]

5) a) Form the Partial Differential Equation by eliminating the arbitrary functions from
$xyz = f(x^2 + y^2 + z^2)$ [6M]

b) Form the Partial Differential Equation by eliminating the arbitrary functions from
$z = xy + f(x^2 + y^2)$ [6M]

5) a) Form the P.D.E by eliminating the arbitrary function from $\emptyset\left(\frac{y}{x}, x^2 + y^2 + z^2\right) = 0$. [6M]

b) Form the P.D.E by eliminating the arbitrary function from $f(x^2 + y^2, z - xy) = 0$. [6M]

6) a) Solve $\frac{\partial^3 z}{\partial x^2 \partial y} + 18xy^2 + \sin(2x - y) = 0$ by direct integration. [6M]

b) Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$. given that when $x = 0, z = e^y$ and $\frac{\partial z}{\partial x} = 1$. [6M]

7) a) Solve $\frac{y^2 z}{x} p + xzq = y^2$. [6M]

b) Solve $(z - y)p + (x - z)q = y - x$. [6M]

8) a) Solve $p(1 + q) = qz$. [6M]

b) Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$. [6M]

9) a) Solve by the method of separation of variables $u_x = 2u_y + u, where\ u(x,0) = 6e^{-3x}$ [6M]

b) Solve by the method of separation of variables $4u_x + u_y = 3u, given\ \ u(0,y) = e^{-5y}$ [6M]

10) a) Solve by the method of separation of variables $3u_x + 2u_y = 0, where\ u(x,0) = 4e^{-x}$ [6M]

b) Solve by the method of separation of variables $u_x - 4u_y = 0, where\ u(0,y) = 8e^{-3y}$ [6M]

UNIT –IV
(Vector Differentiation)

1) a) Find $grad\ f$ if $f = xz^4 - x^2y$ at a point $(1,-2,1)$ .Also find $|\nabla f|$ [6M]

b) If $\bar{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then prove that $\nabla r = \frac{\bar{r}}{r}$ [6M]

2) a) Find the directional derivative of $2xy + z^2$ at $(1,-1,3)$ in the direction of $\vec{i} + 2\vec{j} + 3\vec{k}$. [6M]

b) Find the directional derivative of $xyz^2 + xz$ at $(1,1,1)$ in the direction of normal to the surface $3xy^2 + y = z$ at $(0,1,1)$. [6M]

3) a) Evaluate the angle between the normals to the surface $xy = z^2$ at the points $(4,1,2)$ *and* $(3,3,-3)$. [6M]

b) Find the maximum or greatest value of the directional derivative of $f = x^2yz^3$ at the point $(2,1,-1)$. [6M]

4) a) Find the divergence of $\bar{f} = (xyz)\vec{i} + (3x^2y)\vec{j} + (xz^2 - y^2z)\vec{k}$. [6M]

b) Show that $\bar{f} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x - 2z)\vec{k}$ is *solenoidal.* [6M]

5) a) Find $div\bar{f}$ if $\bar{f} = grad(x^3 + y^3 + z^3 - 3xyz)$. [6M]

b) Find the $curl$ of the vector $\bar{f} = (x + y + 1)\vec{i} + \vec{j} - (x + y)\vec{k}$. [6M]

6) a) Prove that $\bar{f} = (y + z)\vec{i} + (z + x)\vec{j} + (x + y)\vec{k}$ is *irrotational.* [6M]

b) Find $curl\ \bar{f}$ if $\bar{f} = grad(x^3 + y^3 + z^3 - 3xyz)$. [6M]

7) a) Find ‘a’ if $\bar{f} = y(ax^2 + z)\vec{i} + x(y^2 - z^2)\vec{j} + 2xy(z - xy)\vec{k}$ is *solenoidal.* [6M]

b) If $\bar{f} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational then find the constants *a,b and c.* [6M]

8) a) Find $\nabla \times (\nabla \times \bar{f})$, if $\bar{f} = (x^2y)\vec{i} - (2xz)\vec{j} + (2yz)\vec{k}$. [6M]

b) Prove that $div(curl\bar{f}) = 0$. [6M]

9) a) Prove that $\nabla(r^n) = n\, r^{n-2}\bar{r}$ [6M]

b) Prove that $curl(\emptyset\bar{f}) = (grad\emptyset) \times \bar{f} + \emptyset(curl\bar{f})$ [6M]

10) a) Prove that $\nabla.(\bar{f} \times \bar{g}) = \bar{g}.(\nabla \times \bar{f}) - \bar{f}.(\nabla \times \bar{g})$ [6M]

b) Prove that $\nabla \times (\bar{f} \times \bar{g}) = \bar{f}(\nabla.\bar{g}) - \bar{g}(\nabla.\bar{f}) + (\bar{g}.\nabla)\bar{f} - (\bar{f}.\nabla)\bar{g}$ [6M]

UNIT –V
(Vector Integration & Integral theorems)

1) a) If $\bar{F} = (5xy - 6x^2)\vec{i} + (2y - 4x)\vec{j}$ .Evaluate $\int_c \bar{F}.d\bar{r}$ along the curve 'c' in xy-plane $y = x^3$ from $(1,1) to (2,8)$. [6M]

b) Find the work done by a force $\bar{F} = (2y + 3)\vec{i} + (xz)\vec{j} + (yz - x)\vec{k}$ when it moves a particle from $(0,0,0) to (2,1,1)$ along the curve $x = 2t^2; y = t\,; z = t^3$. [6M]

2) If $\bar{F} = (x^2 + y^2)\vec{i} - (2xy)\vec{j}$. Evaluate $\int_c \bar{F}.d\bar{r}$ where 'c' is the rectangle in xy-plane bounded by $y = 0; y = b\ and\ x = 0; x = a$. [12M]

3) a) Evaluate $\int_s \bar{F}.\bar{n}ds$. where $\bar{F} = 18z\vec{i} - 12\vec{j} + 3y\vec{k}$ and 's' is the part of the surface of the plane $2x + 3y + 6z = 12$ located in the first octant. [6M]

b) Evaluate $\int_s \bar{F}.\bar{n}ds$. where $\bar{F} = 12x^2y\vec{i} - 3yz\vec{j} + 2z\vec{k}$ and 's' is the portion of the plane $x + y + z = 1$ located in the first octant. [6M]

4) a) If $\bar{F} = 2xz\vec{i} - x\vec{j} + y^2\vec{k}$. Evaluate $\int_v \bar{F}.dv$ where 'v' is the region bounded by the surfaces $x = 0; x = 2 : y = 0; y = 6\ and\ z = x^2; z = 4$. [6M]

b) If $\bar{F} = (2x^2 - 3z)\vec{i} - 2xy\vec{j} - 4x\vec{k}$ then Evaluate $\int_v \nabla.\bar{F}\ dv$ where 'v' is the closed region bounded by $x = 0; y = 0; z = 0\ and\ 2x + 2y + z = 4$. [6M]

5) a) State Gauss's divergence theorem. [2M]

b) By transforming into triple integral, Evaluate $\iint_s x^3dydz + x^2ydzdx + x^2zdxdy$ where 's' is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$ and the circular discs $z = 0; z = b$. [10M]

6) Verify Gauss's divergence theorem for $\bar{F} = (x^3 - yz)\vec{i} - 2x^2y\vec{j} + z\vec{k}$ taken over the surface of the cube bounded by the planes $x = y = z = a$ and coordinate planes. [12M]

7) a) Apply Green's theorem to Evaluate $\oint_c (2x^2 - y^2)dx + (x^2 + y^2)dy$ where 'c' is the enclosed by the x-axis and upper half of the circle $x^2 + y^2 = a^2$. [6M]

b) Evaluate by Green's theorem $\oint_c (y - sinx)dx + cosxdy$ where 'c' is the triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}\ and\ \pi y = 2x$. [6M]

8) a) State Green's theorem in a plane. [2M]

b) Verify Green's theorem in a plane for $\oint_c (x^2 - xy^3)dx + (y^2 - 2xy)dy$ where 'c' is a square with vertices $(0,0)(2,0)(2,2)\ and\ (0,2)$. [10M]

9) Verify Stoke's theorem for $\bar{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken round the rectangle bounded by the lines $x = \pm a\ ,\ y = \pm b$. [12M]

10) a) State Stoke's theorem. [2M]

b) Verify Stoke's theorem for $\bar{F} = x^2\vec{i} + xy\vec{j}$ integrated round the square in the plane $z = 0$,whose sides are along the line $x = 0, y = 0\ ; x = a, y = a$. [10M]